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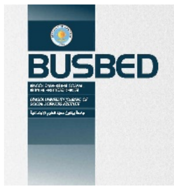

ESTIMATION OF EFFECT ON GROSS DOMESTIC PRODUCT OF PRODUCTION FACTORS USING CES AND TRANSLOG PRODUCTION FUNCTIONS: AN APPLICATION TO CHINA ECONOMY

Ömer ÖNALAN[1], Hülya BAŞEĞMEZ[2]

ABSTRACT

In this study, the effects on the economic growth (GDP) of capital, labor and energy input factors for the Chinese economy are investigated with the help of the CES and Translog production functions. According to the empirical findings of the study, it can be said that the GDP data obtained using the CES production function are less efficient than the estimates obtained using the Translog production function. The Ridge regression technique was used for parameter estimation of Translog production model since there is multicollinearity between the variables in the model. The output elasticities and the substitution elasticities between each input factor are then dynamically estimated, based on the appropriate Translog production model that includes the capital, labor and energy input factors. In addition, the inputs of the Translog production model were estimated using the Holt-Winter's method to predict the future economic growth of the Chinese economy. Consequently, output elasticities of all input factors are positive, and we can rank the input factors as labor, capital and energy according to their degree of impact on GDP, respectively. This shows that the Chinese economy is labor and capital intensive.

Keywords: Gross Domestic Product, CES Production Function, Translog Production Function, Holt-Winter's Method, Ridge Regression.

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CES VE TRANSLOG ÜRETİM FONKSİYONLARI KULLANILARAK ÜRETİM FAKTÖRLERİNİN GDP ÜZERİNDEKİ ETKİSİNİN KESTİRİMİ: ÇİN EKONOMİSİ İÇİN BİR UYGULAMA

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ÖZ

Bu çalışmada, CES üretim fonksiyonu ve Translog üretim fonksiyonunu yardımıyla Çin ekonomisine ait sermaye, emek ve enerji girdi faktörlerinin ekonomik büyüme (GSYİH) üzerindeki etkisi araştırılmaktadır. Çalışmanın ampirik bulgularına göre, CES üretim fonksiyonu kullanılarak elde edilen GSYİH verilerinin Translog üretim fonksiyonu kullanılarak elde edilen kestirimlerden daha az verimli olduğu söylenebilir. Modelde yer alan değişkenler arasında çoklu doğrusal bağıntı problemi mevcut olduğu için parametre kestiriminde Ridge regresyon tekniği kullanılmıştır. Daha sonra sermaye, emek ve enerji girdi faktörlerini içeren uygun Translog üretim modeline dayalı olarak, faktörlerin her biri için çıktı esnekliklerinin sonuçları ve girdi faktörleri arasındaki ikame esnekliklerinin sonuçları dinamik olarak tahmin edilmiştir. Ayrıca, Çin ekonomisinin gelecekteki ekonomik büyümesini tahmin etmek için Translog üretim modelinin girdileri Holt-Winter's yöntemi kullanılarak tahmin edilmiştir. Tüm girdi faktörlerinin çıktı esneklikleri pozitif olup, girdi faktörlerini GSYİH üzerindeki etki derecelerine göre, sırasıyla, emek, sermaye ve enerji olarak sıralayabiliriz. Bu durum Çin ekonomisinin emek ve sermaye yoğun olduğunu göstermektedir.

Anahtar Kelimeler: Gayri Safi Yurtiçi Hasıla, CES Üretim Fonksiyonu, Translog Üretim Fonksiyonu, Holt-Winter's yöntemi, Ridge regresyon

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1. INTRODUCTION

The production process transforms the inputs into outputs. Gross Domestic Product (GDP) in general economy can be described by a production function. Thus, the determination of the functional form of the production function is an important issue in the empirical analysis. There are four different production function models commonly used in the literature. These are the Cobb-Douglas production function for which elasticity of substitution is equal to one, Constant Elasticity of Substitution (CES) production function, Leontief production function and Translog production function. In this study, we used CES production function and Translog production function to establish the impact of capital investments, labor and energy on economic development.

Analyzing the factor substitution is very important to develop economic policies. In this study, firstly, we used the classic assumption that the elasticities of substitutions are constant and the CES model. But in fact, this assumption may not be realistic. It may change over time. Secondly, we used the Translog production function as a model for variable elasticity substitution. In this model, it is avoided using the data of factor prices.

CES production function has a lot of applications in macroeconomics as international (Lloyd & MacLaren, 2002), economic growth (Papageorgiou & Saam, 2008) and energy economics (McFarland et al., 2004). CES functions are non-linear and they can also not be linearized. So, conventional linear estimation methods cannot be implemented for CES production functions. There are some difficulties in estimating the parameters of the CES production function, such as the surface of the objective function having a large flat area and local minimum. The popular parameter estimation methods for CES production function are linear Taylor series approximation and non-linear least squares method (Henningsen & Henningsen, 2012). In addition, Henningsen, Henningsen and Van Der Werf (2019) advised against the use of direct estimation methods in empirical applications in the absence of long time series and high independent variation of the inputs. Lagomarsino (2020), critically examined the types of nesting structures, estimation approaches, data sources, the types of substitution elasticities used by researchers working on CES production function in the literature.

Translog production function was evaluated as an extension of Cobb – Douglas production function by Christensen et al. (1973). Then, Romer (1986), Lucas Jr. (1988), Mankiw et al. (1992) and Benhabib and Spiegel (1994) have made important contributions to the literature on the Translog production function. The translog model has a more flexible form than the Cobb Douglas production function as it provides a local, quadratic approximation to any function, but it is more problematic to estimate due to the large number of parameters and the possibility of multicollinearity between regressors (Irz & Mckenzie, 2008). The most widely used method for parameter estimation of the Translog production function is the Ridge regression method which is also used in this study. Songur (2017) estimated the output elasticity of human capital and physical capital in Turkey and the elasticity of substitution between these factors using the Ridge Regression method. Şanli and Konukman (2021) used the Translog production function to measure the impact of high-tech exports on income. Wei and Wang (2021) made an estimation of the production function in the context of Stochastic frontier analysis, using the Translog production function to measure the effect of industrial air pollution on agricultural production.

There are many factors that have contributed to GDP, such that capital, labor, energy, optimal allocation of technology sources, innovations, etc. In this study, CES production function and Translog production function models are used to confirm that capital, labor, and energy have an important effect on GDP. When examining the applications of production function models with actual data, it is generally seen that multiple regression models are used. In a multiple regression application, we need to know the future values of the inputs (independent variables) to the prediction of future values of the response variable. Thus, future values of capital, labor and energy input factors are also forecasted based on Holt-Winter's method in this study. In this regard, the study differs and contributes to the literature.

The aim of this paper is twofold. Firstly, it is to investigate the output elasticity for each input factor and the elasticity of substitution between capital, labor and energy input factors. Secondly, it is to estimate the future values of GDP which is an indicator of China's Economic growth.

The CES production function and Translog production function are implemented in real data. To predict the future values of GDP, capital, labor and energy input factors are predicted using by Holt-Winter's method, then based on the fitted model is analyzed the variation of the elasticities according to levels of input factors within the considered time period.

The remaining part of this study is organized as follows. Section II reviews the fundamental properties of CES production function, nested CES function and parameter estimation methods. Section III describes the model framework and the estimation procedures such as Translog production function, Ridge regression method and Holt-Winter's method. In Section IV, we describe the data set, how the data is processed and it is presented the empirical results and discussion. Section V includes the conclusion of the study.

2. CONSTANT ELASTICITY OF SUBSTITUTION (CES) PRODUCTION FUNCTION

GDP can be considered as a measure of the economic growth of countries. The main factor affecting the GDP is industrial output. The industrial output can be considered as a function of capital and labor inputs (Hossain et al., 2013). In substance, economics production is affected by various environmental factors as capital, labor, agricultural activities, technology, industry, energy, raw materials, etc. Production is a process that converts production factors (inputs) into finished products (outputs).

In the economics literature, the empirical relationship between given the quantity of economics inputs and specified outputs is represented as the production function. One of the main problems for economic governance in the production process is determination of the functional relationship between the production output and input factors. The general form of the production function is described by $f: D \to R_+$, $D \in R_+^n$ and $Q = f(X_1, X_2, \dots, X_n)$ where $X_1,\ X_2, \dots, X_n$ are inputs and Q is production level. This function is a differentiable function of all its inputs. Variables are mostly available capital stock in terms of the value of machinery and buildings and labor in terms of people employed or land in terms of acres. Therefore, it is assumed that the inputs of this function are homogeneous. The production function with n input factors is called $h-$homogeneous degree, if

$$f(kx_1, kx_2, \dots, kx_n) = k^h f(x_1, x_2, \dots, x_n), \qquad (h > 0)$$

where $k \epsilon R$ and if $h > 1$, per percent increase in input levels would result greater than per percent increase in the output level, if $h < 1$, per percent increase in input levels would result in less than per percent increase in output and $h = 1$ represent the constant return to scale.

The production functions are characterized according to technology elasticity of substitution and return to scale. Hicks elasticity of substitution measure to a production function with more than two inputs is described as (Stern, 2011),

$$\sigma_{ij} = -\frac{\partial \ln\left(X_i/X_j\right)}{\partial \ln\left((\partial Q/\partial X_i)/\left(\partial Q/\partial X_j\right)\right)}.$$

2.1. Functional Form of CES Production Function

The Constant - Elasticity of Substitution (CES) production function is originally given by Arrow, Chenery, Minhas and Solow (1961). It is a generalization of the Cobb-Douglas production function. The CES function puts a restriction on elasticity substitution of being constant along the whole isoquant. The CES production function is based on the assumption that the elasticities of substitution between any two inputs are the same.

The CES production function has many forms. In this study, we will use the CES production function proposed by Kmenta with capital and labor inputs (Kmenta, 1967). It is given as follows,

$$Q = A[\delta K^{-\rho} + (1-\delta)L^{-\rho}]^{-\vartheta/\rho} \tag{1}$$

where Q denotes the total value of output, L denotes input of labor which is measured in men-per (and hour) year, K represents the capital input which is measured in money term, $A \in [0,\infty)$ represents the productivity (technologic progress level), $\delta \in [0,1]$ denotes the inputs' optimal distribution, $\rho \in [-1,0) \cup (0,\infty)$ represents the elasticity of substitution and $\vartheta \in (0,\infty)$ ($\vartheta = K/L > 0$) is the function's homogenous order or rate of return to scale (degree of homogeneity). In the original form of CES, parameter ϑ was taken as $\vartheta = 1$. If $\vartheta = 1$, constant return to scale, if $\vartheta < 1$, decreasing return to scale and if $\vartheta > 1$, increasing return to scale.

The elasticity of substitution (EOS) of CES function is written as,

$$\sigma = \frac{\partial \ln(K/L)}{\partial \ln(MP_L/MP_K)} = \frac{1}{1+\rho} \geq 0. \tag{2}$$

The logarithmic form of CES function is given by

$$\ln Q = \ln\gamma - \frac{\vartheta}{\rho}\ln[\delta K^{-\rho} + (1-\delta)L^{-\rho}]. \tag{3}$$

Uzawa (1962) and McFadden (1963) tried to extent the CES function to $n-$input factor production functions. Marginal productivity of labor for CES function is given by

$$\frac{\partial Q}{\partial L} = (1-\delta)\,A^{-\rho}\left(\frac{Q}{L}\right)^{\rho+1}.$$

Then,

$$\frac{Q_L}{Q} = (1-\delta)A^{-\rho}\left(\frac{Q}{L}\right)^{\rho+1}. \tag{4}$$

In Eq. (4), taking the logarithm of both side, we obtained

$$\ln(Q_L/Q) = \ln\big((1-\delta)A^{-\rho}\big) + (\rho+1)\ln(Q/L). \tag{5}$$

Eq. (5) represents the relationship between nominal wage, the market price of production and average production.

2.2. Parameter Estimation of CES Production Function

CES production function is nonlinear in parameters, so it cannot be linearized to estimate the parameters using the traditional linear estimation methods. In the parameter estimation of CES production function is used to non-linear fitting techniques. In the estimation process, we assume that input variables are non-stochastic or if they are stochastic, independent of disturbance term (Hoff, 2002).

In general, two approaches widely used in CES parameter estimation are linear Taylor series approach and non-linear least squares method. We can use a linear approach with respect to ρ.

2.3. Estimating the CES function using Kmenta Approximation

$\ln Q$ series expanding the Taylor series around the $\rho = 0$ and discarding the terms of third and higher order expanding leads the following form (Kmenta, 1967),

$$\ln Q_i = \ln\gamma + \vartheta\delta\ln K_i + \vartheta(1-\delta)\ln L_i - \left(\frac{1}{2}\right)\rho\vartheta\delta(1-\delta)[\ln K_i - \ln L_i]^2 + \varepsilon_i \tag{6}$$

Let we define the new variables as,

$$Y^* = \ln Q_i\,, \qquad X_1^* = \ln K_i\,, \qquad X_2^* = \ln L_i\,, \qquad X_3^* = [\ln K_i - \ln L_i]^2,$$

$$\beta_0 = \ln\gamma\,, \quad \beta_1 = \vartheta\delta\,, \quad \beta_2 = \vartheta(1-\delta)\,, \quad \beta_3 = -\left(\tfrac{1}{2}\right)\rho\vartheta\delta(1-\delta).$$

Then, Eq. (6) can be written as follows according to this new parametrization.

$$Y^* = \beta_0 + \beta_1 X_1^* + \beta_2 X_2^* + \beta_3 X_3^* + \varepsilon. \tag{7}$$

The application of the ordinary least squares method yields the following parameter estimations (Mahaboob et al., 2017),

$$\hat{\gamma} = e^{\hat{\beta}_0}, \quad \hat{\delta} = \frac{\hat{\beta}_1}{\hat{\beta}_1+\hat{\beta}_2}\,, \quad \hat{\vartheta} = \hat{\beta}_1 + \hat{\beta}_2\,, \quad \hat{\rho} = -2\left(\frac{\hat{\beta}_1+\hat{\beta}_2}{\hat{\beta}_1\hat{\beta}_2}\right)\hat{\beta}_3 \tag{8}$$

The disadvantage of this approach is that it gives reliable results if ρ is close the point zero (Thursby & Lovell, 1978).

The substitution elasticity of (EOS) σ is a function of ρ and estimated as,

$$\hat{\sigma} = \frac{1}{1+\hat{\rho}}.$$

2.4. Nested CES Production Function

A nested production function is given by Henningsen et al. (2019) and Kemfert (1998),

$$Q_t = e^{mt}A\left[a(bK_t^{-\alpha} + (1-b)E_t^{-\alpha})^{\beta/\alpha} + (1-a)L_t^{-\beta}\right]^{-\gamma/\beta}$$

where a and b denote the share parameters denoting input factor contributions to output. α and β denote the substitution parameters, A ≥ 0 represents the productivity (efficient), m denotes the annual rate of technological change, t is the time in years and Q denotes the output. In the single level CES function, all input factors have the same substitution elasticity with each other. However, the nested CES function allows the different substitution elasticities between the factors.

When the model is applied the real data, firstly capital and energy combined into an intermediate input using a two-input CES function with share parameter b and substitution parameter α. Secondly, this intermediate input is combined with labor L into output Q using another CES function with share parameter a and substitution parameter β.

$$Q_t = Ae^{mt}X^{-\gamma/\beta}$$

$$X = aY^{\beta/\alpha} + (1-a)L^{-\beta}$$

$$Y = bK^{-\alpha} + (1-b)E^{-\alpha}$$

$$\rho_{in} = \frac{1}{1+\alpha} \quad \text{and} \quad \rho_{out} = \frac{1}{1+\beta}$$

ρ_{in} and ρ_{out} are inner and outer nested substitution parameters, respectively.

The parameters of nested CES function is estimated using non-linear least squares method as,

$$\min_{A,m,a,b,\alpha,\beta} \sum_t \left(\ln Q_t - mt - \ln A + \frac{1}{\beta}\ln\left[a(bK_t^{-\alpha} + (1-b)E_t^{-\alpha})^{\beta/\alpha} + (1-a)L_t^{-\beta}\right]\right)^2.$$

To estimate the parameters of CES function, Kemfert (1998) is used the statistical software SHAZAM.

3. TRANSLOG PRODUCTION FUNCTION

The Translog function belongs to the quadratic response surface methodology. The transcendental logarithmic function (Translog production function) was first introduced by (Christensen et al., 1973). The Translog production function is a more general form of variable elasticity of substitution. It does not impose any restrictions on the substitutability between different input factors. A Translog production function can be view as a second-order Taylor approximation for arbitrary function (Lin & Liu, 2017). The Translog function is widely used to detect the substitution effect of explanatory variables. The interactions between the components of interested system may be described as a quadratic response surface as follows,

$$\hat{y} = a_0 + a_1x_1 + a_2x_2 + a_3x_3 + a_{12}x_1x_2 + a_{12}x_1x_2 + a_{13}x_1x_3 + a_{23}x_2x_3 + a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2$$

Generally, a Translog production function is described as follows;

$$\ln Q_t = \beta_0 + \sum_i \beta_i \ln X_i + \sum_i \beta_{ii}(\ln X_i)^2 + \frac{1}{2}\sum_{i\neq j} \beta_{ij}(\ln X_i)(\ln X_j) + \varepsilon \tag{9}$$

Using the symmetry property of partial derivatives, i.e. $\frac{1}{2}\left(\beta_{ij} + \beta_{ji}\right) = \beta_{ij}$, we will use the following form of the Translog production function in practice,

$$\ln Q_t = \beta_0 + \sum_{i=1}^{n} \beta_i \ln X_i + \sum_{i=1}^{n} \beta_{ii}(\ln X_i)^2 + \sum_{j>1}^{n} \beta_{ij}(\ln X_i)(\ln X_j) + \varepsilon \tag{10}$$

where Q is the output, X_i are inputs and β_0, β_i and β_{ij} are parameters to be estimated. Parameter β_0 denotes the technology level.

Generally, response surface function with $n-$ variables is estimated as follows. Polynomial coefficients are determined based on least squares regression method. We assume that there are k observation points.

$$\boldsymbol{Q} = \begin{bmatrix} \ln Q_1 \\ \ln Q_2 \\ \vdots \\ \ln Q_k \end{bmatrix}$$

$$\boldsymbol{X} = \begin{bmatrix} 1 & \ln X_{11} & \cdots & \ln X_{1n} & (\ln X_{11})^2 & \cdots & (\ln X_{1n})^2 & \ln X_{11}\ln X_{12} & \cdots & \ln X_{1n}\ln X_{1n-1} \\ 1 & \ln X_{21} & \cdots & \ln X_{2n} & (\ln X_{21})^2 & \cdots & (\ln X_{2n})^2 & \ln X_{21}\ln X_{22} & \cdots & \ln X_{2n}\ln X_{2n-1} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & \ln X_{k1} & \cdots & \ln X_{kn} & (\ln X_{k1})^2 & \cdots & (\ln X_{kn})^2 & \ln X_{k1}\ln X_{k2} & \cdots & \ln X_{kn}\ln X_{kn-1} \end{bmatrix}$$

$$\boldsymbol{\beta} = [\beta_0 \quad \beta_1 \quad \cdots \quad \beta_n \quad \beta_{11} \quad \cdots \quad \beta_{nn} \quad \beta_{12} \quad \cdots \quad \beta_{n,n-1}]^T$$

$$\boldsymbol{\varepsilon} = [\varepsilon_2 \quad \varepsilon_2 \quad \cdots \quad \varepsilon_2]^{\boldsymbol{T}}$$

$$\boldsymbol{Q} = \boldsymbol{X\beta} + \boldsymbol{\varepsilon}$$

$$\hat{\beta} = (\boldsymbol{X}^T\boldsymbol{X})^{-1}\boldsymbol{X}^T\boldsymbol{Q}$$

The elasticity of output with respect to input X_i is given as,

$$f_i = \frac{\partial Q}{\partial X_i}\frac{X_i}{Q} = \beta_i + \sum_{j=1}^{n} \beta_{ij} \ln X_j \tag{11}$$

The marginal product of input X_i is given as,

$$y_i = \frac{\partial Q}{\partial X_i} = \left[\beta_i + \sum_{j=1}^{n} \beta_{ij} \ln X_j\right] . \frac{Q}{X_i} \tag{12}$$

A Translog production function with $(n = 3)$ three input factors $Q = f(K, L, E)$ which includes the capital stock, labor force and energy consumption is defined as;

$$\begin{aligned} \ln(Q_t) = \beta_0 + \beta_K \ln K_t + \beta_L \ln L_t + \beta_E \ln E_t + \beta_{KK}(\ln K_t)^2 + \beta_{LL}(\ln L_t)^2 + \beta_{EE}(\ln E_t)^2 \\ + \beta_{KL}(\ln K_t \ln L_t) + \beta_{KE}(\ln K_t \ln E_t) + \beta_{LE}(\ln L_t \ln E_t) \end{aligned} \tag{13}$$

In order for a production function to have consistent and desired properties the following three conditions are required, i) monotonically increasing, ii) continuity, iii) exact concave. A Translog function generally satisfies these conditions. The exact concavity property of Translog function is tested using the Hessian matrix.

The output elasticities with respect to input factors are as follows:

The output elasticity of Capital,

$$f_K = \frac{dQ/Q}{dK/K} = \beta_K + \beta_{KL} \ln L_t + \beta_{KE} \ln E_t + 2\beta_{KK} \ln K_t$$

The output elasticity of Labor force,

$$f_L = \frac{dQ/Q}{dL/L} = \beta_L + \beta_{LK} \ln K_t + \beta_{LE} \ln E_t + 2\beta_{LL} \ln L_t$$

The output elasticity of Energy,

$$f_E = \frac{dQ/Q}{dE/E} = \beta_E + \beta_{KE} \ln K_t + \beta_{LE} \ln L_t + 2\beta_{EE} \ln E_t \,.$$

The elasticity of substitution is described by the ratio between the percentage changes in the proportion of input factors. Substitution elasticity indicates how hard to substitute one input factor with the other. It is varying zero to infinity. The substitution elasticity between different factors is given as Lin and Ahmad (2016) and Lin and Liu (2017),

The substitution elasticity between Capital and Labor,

$$\sigma_{KL} = \left[1 + \left[-\beta_{KL} + \left(\frac{f_K}{f_L}\right)\beta_{LL}\right](-f_K + f_L)^{-1}\right]^{-1}$$

The substitution elasticity between Capital and Energy,

$$\sigma_{KE} = \left[1 + \left[-\beta_{KE} + \left(\frac{f_K}{f_E}\right)\beta_{EE}\right](-f_K + f_E)^{-1}\right]^{-1}$$

The substitution elasticity between Labor and Energy,

$$\sigma_{LE} = \left[1 + \left[-\beta_{LE} + \left(\frac{f_L}{f_E}\right)\beta_{EE}\right](-f_L + f_E)^{-1}\right]^{-1}.$$

3.1. Estimation of Translog Production Function

The Translog function can be considered as a second-order Taylor approximation of an arbitrary function. It is also similar to a linear regression model according to coefficients. Thus, the parameters of Translog production function in Eq. (13) can be estimated using the Ordinary Least Square (OLS) method.

3.2. Ridge Regression

The linear regression model is given by,

$$Y = \beta X + e$$

where Y is a vector of the dependent variable, X describes $n \times p$ explanatory variables. e is random error. The assumptions of regression model are that $E(e) = 0,\ \ Var(e) = \sigma^2 I_n,\ e$ is a random variable that has a normal distribution. There is not a relationship between independent variables and residuals.

The collinearity phenomena occur when there is linear or approximately linear relationship between two or more independent variables. In the existence of collinearity in the regression model, The OLS method is to be biased and singular. To obtain sensitive parameter estimation, the collinearity must be eliminated. In this study, the independent variables without removing from the model, to eliminate the collinearity, we will use the ridge regression estimator. We may detect the existing multicollinearity using one of the following methods (NCSS Statistical Software, 2019):

(1) Correlation coefficient: If the correlation coefficient $r > 0{,}75$, we suspect from the collinearity.
(2) Condition numbers: The eigenvalues of the correlation matrix created for the independent variables are close to zero, indicating multicollinearity. The condition number can be used instead of the numerical values of the eigenvalues. If the condition numbers are large, multicollinearity exists.
(3) Variance Inflation Factor (VIF): VIF value is calculated as,

$$C_{ij} = \frac{1}{1 - R_{ij}}$$

where, R_{ij} is partial correlation. If $C_{ij} > 10$, there is collinearity.

Ridge regression is a biased regression estimation method. Ridge regression estimator has proposed by Hoerl & Kennard (1970). Ridge regression method is used to eliminate the collinearity problem in data analysis. Ridge regression is an improvement version of OLS method.

In the presence of multicollinearity, a constant small value of k is added to the diagonal elements in correlation matrix $(X'X)$ to reduce the dependence on independent variables. Ridge estimator is given by,

$$\hat{\beta}(k) = (X'X + kI)^{-1}X'Y \tag{14}$$

where k $(0 < k < 1)$ is ridge parameter (shrinkage parameter) and I is the identity matrix. When $k = 0$ the ridge estimator $\hat{\beta}$ is the OLS estimation (Hrishikesh D. Vinod, 1978). Covariance matrix for ridge estimator is,

$$Var(\hat{\beta}) = \sigma^2 (X'X + kI)^{-1} X'X (X'X + kI)^{-1}$$

where $\hat{\sigma}^2 = \frac{1}{n-p}\sum_{i=1}^{n} e_i^2$ and $e_i = y_i - \hat{y}_i$. Total MSE of the ridge estimator is given by (Hoerl & Kennard, 1970).

$$MSE(\hat{\beta}) = \sigma^2 \sum_{i=1}^{p} \lambda_i (\lambda_i + k)^{-1} + k^2 \beta' (X'X + kI)^{-2} \beta$$

where λ_i are the eigenvalue of $(X'X)$ correlation matrix. The choice of appropriate k value (a) the regression coefficients must be stable, (b) Variance Inflation Factor (VIF) must be small. In the literature, there are different methods to obtain optimal value of ridge parameter k. The most popular of them is that the $\hat{\beta}(k)$ values are plotted with respect to the values of k, then the value of $\hat{\beta}(k)$ that is stable is selected as the optimal ridge parameter k.

Let the eigenvalues of $(X'X)$ correlation matrix $\lambda_{\max} = \lambda_1 > \lambda_2 > \cdots > \lambda_p = \lambda_{min}$, then

$$k \leq \frac{\lambda_{max} - 100\lambda_{min}}{99}$$

To estimate the value of k is given by Muniz and Kibria (2009),

$$k = \left(\prod_{i=1}^{p} \frac{1}{m_i}\right)^{1/p} \quad , \quad m_i = \left(\frac{\hat{\sigma}^2}{\hat{\beta}_i^2}\right)^{1/2} .$$

VIF show how the variance of an estimator is inflated by the presence of multicollinearity. When the variance of an estimator R_{ij} approaches 1, collinearity increases. Any value of VIF that is greater than 10 strongly indicates the presence of collinearity.

3.3. Holt-Winter's Method

We will use the Holt-Winter's exponential smoothing method to get the basic forecast values of the Capital, Labor and Energy input variables. This method is a common seasonal forecasting method. The multiplicative Holt-Winter's method is described following three basic equations Dhakre et al. (2016), Hamilton J. D. (1994) and Prado and West (2010),

Exponential trend for long-term trend : $S_t = \alpha \left(\frac{y_t}{I_{t-p}}\right) + (1-\alpha)(S_{t-1} + b_{t-1})$

Trend smoothing : $b_t = \beta(S_t - S_{t-1}) + (1-\beta)b_{t-1}$

Seasonal smoothing : $I_t = \gamma \left(\frac{y_t}{S_t}\right) + (1-\gamma)I_{t-p}$

Forecasting : $F_{t+m} = (S_t + b_t)I_{t+m-p}$

where α, β and γ are parameters takes the values in interval $[0,1]$, t is the time period, y_t is observed value of series at time t.

4. EMPIRICAL ANALYSIS

In this study, we use the annual GDP series as a proxy of aggregate national economic level for China economy. We estimated the future economic growth pattern of China by focusing on the 1994-2014 period. We use the annual data gross domestic product Q (0.1 billion yuan) as an output, fixed asset investment K (0.1 billion yuan), the number of employees L (10,000 people) and total energy consumption E (10,000 tons of standard coal) as input factors to analyze the China economy between 1994-2014. Data are obtained from the World Bank's World development indicators (WDI) China Statistical Yearbook and China Energy Yearbook (CSYD) as published by China's National Bureau of Statistic and by (Cheng & Han, 2017).

It was estimated the future values of GDP which is a function of capital, labor and energy input factors by using Improved GM (1,1) Model and Grey CES production function. Firstly, we investigated the effect on the economic growth of capital and labor by using CES production function. The model parameters are estimated using Kmenta approach and we predicted the economic growth.

Capital (K_t): No capital stock data is reported in the China statistical system. The time series of capital is usually not found in economics databases. It is obtained from other sources. Capital is calculated using the perpetual inventory method (Raymond W. Goldsmith, 1951).

$$K_t = I_t + (1 - \delta_t)K_{t-1}$$

where K_t is capital stock in year t, I_t is investment in year t. Initial capital stock K_0 is determined as follows,

$$K_0 = \frac{I_0}{\overline{GDP} + 0{,}05}$$

Labor (L): The number of employees. Data obtained from China Statistical Yearbook.

Energy consumption (E_t): Energy consumption data taken from China Statistical Yearbook and APEC energy database.

4.1. Parameter Estimation of CES Production Function

The regression estimation results of CES production function is given in Table1.

Table 1.
The regression estimation results

	Coefficients	Standard Error	t-Stat	P-value
Intercept	-35.987	13.039	-2.760	0.013
$\ln K$	0.575	0.051	11.228	0.000
$\ln L$	3.708	1.213	3.058	0.007
$\left(\ln\left(\frac{K}{L}\right)\right)^2$	0.019	0.021	0.939	0.361
Adjusted R-Square = 0.997, F-statistics = 2213.223				

Table 2.
Parameter values of CES production function

$\boldsymbol{\delta}$	$\boldsymbol{1-\delta}$	$\boldsymbol{\theta}$	$\boldsymbol{\rho}$	$\boldsymbol{\sigma}$	$\boldsymbol{\gamma}$	$\boldsymbol{-\theta/\rho}$
0.134	0.866	4.283	-0.078	1.085	$e^{-35.987}$	54.72

Estimated parameter values of CES production function is given in Table 2. Using these values, CES function can be rewritten as,

$$\hat{Q}_{CES} = \ e^{-35.987}[0.134\, K^{0.078} + 0.866\, L^{0.078}]^{54.72}\,. \tag{15}$$

GDP values were estimated using Eq. (15). Accordingly, the actual and estimated values graphic obtained from Kmenta approach for GDP is presented in Figure 1. When Figure 1 is examined, it can be seen that the graph obtained from the estimated GDP values and the graph obtained from actual GDP values fit quite well.

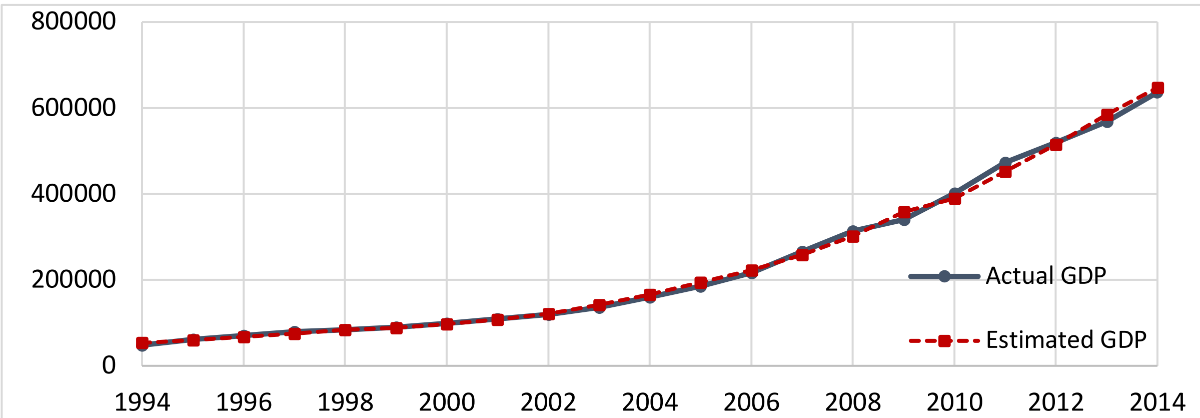

Figure 1. Actual GDP and Estimated GDP obtained from Kmenta Approach

The estimated value of elasticity of substitution $\hat{\sigma}_{CES} = 1.085$ denotes that CES production function may be a suitable model to describe the China economic growth. According to the $\hat{\sigma}_{CES}$ estimation, we can say that the general economy has a substitution potential between capital and labor inputs.

Substituting the input factor that has a high unit price with the input factors that have relatively lower input price, the total production costs can be reduced.

It was previously stated that $\delta \in [0,1]$ parameter denotes the optimal distribution of the inputs. Table 2 shows the values of $\hat{\delta} = 0.134$ and $1 - \hat{\delta} = 0.866$. The values of $\hat{\delta} = 0.134$ and $1 - \hat{\delta} = 0.866$ imply the distribution rate (share) of total output(Q) via capital (values of building, lands, machinery, other tools etc.) and labor (wages and salaries), respectively. Disribution parameter δ reflects capital intensity in production (Klump, Mcadam, & Willman, 2012). According to these results, we can say that the contribution rate of a unit of capital to production is 0.134, and the contribution rate of a unit of labor to production is 0.866.

Parameter $\widehat{\vartheta} = 4.283$ represents the elasticity of scale, this result implies that China economy has increasing return to scale.

4.2. Translog production model

In this section, we want to build a Translog production function with the capital, labor and energy input factors. Due to interaction and squared terms of the input factors in Eq. (13), the model is likely to suffer from severe multicollinearity. So before establishing a regression equation for Translog production function, we must detection the multicollinearity existing (Lin & Xie, 2014).

In this study, NCSS 2007 software was used in parameter estimation process. In order to detect the multicollinearity, the multicollinearity test was firstly performed in the NCSS program. As can be seen from Table 3, the condition number obtained as a result of this test is a very large value, so the existence of a multicollinearity problem is assured. Then, regression analysis was performed for Translog production function.

Table 3.
Multicollinearity Test

Variable	Detection tolerance	R^2	VIF	Multicollinearity problem is present?	Condition Number
GDP	0.3%	99.7%	328.90	True	27112687.43
K	0.7%	99.3%	152.89	True	
L	13.4%	86.6%	7.45	True	
E	1.8%	98.2%	55.64	True	

Table 4 shows the estimation results of the Ridge regression with the Ridge parameter $k = 0.0006$.

Table 4.
Ridge Regression Coefficient for $k = 0.006$

Variables	Coefficient	Std. Error	Standardized Coefficient	VIF
β_0	-22.337470			
$\ln K$	0.145865	0.027855	0.204000	4.372500
$\ln L$	1.748346	0.474985	0.091300	1.771700
$\ln E$	-0.013067	0.089362	-0.007000	6.517000
$\ln K \ln K$	0.007516	0.001731	0.240500	8.840200
$\ln L \ln L$	0.076086	0.021126	0.088900	1.755400
$\ln E \ln E$	0.001052	0.003231	0.013800	5.151300
$\ln K \ln L$	0.012340	0.002304	0.200100	4.022500
$\ln K \ln E$	0.007475	0.001070	0.173800	1.784400
$\ln L \ln E$	0.000447	0.006575	0.002900	5.340500
R-Squared: 0.9962	Sigma : 0.0669			

According to the result of Ridge regression, the R-square fitting degree value of Translog equation is 99.62% and the multicollinearity of variables is effectively eliminated. The coefficients of technological progress input and energy input are negative and other variables have positive coefficients. Since all VIF values are less than 10, multicollinearity is no longer a problem.

According to Table 4, the corresponding Ridge regression equation can be written as follows,

$$\ln \hat{Q}_t = -22.337470 - 0.145865 \ln K_t + 1.748346 \ln L_t + -0.013067 \ln E_t + 0.007516 (\ln K_t)^2 + 0.076086 (\ln L_t)^2 + 0.001052 (\ln E_t)^2 + 0.012340 (\ln K_t \ln L_t) + 0.007475 (\ln K_t \ln E_t) + 0.000447 (\ln L_t \ln E_t) \,.$$

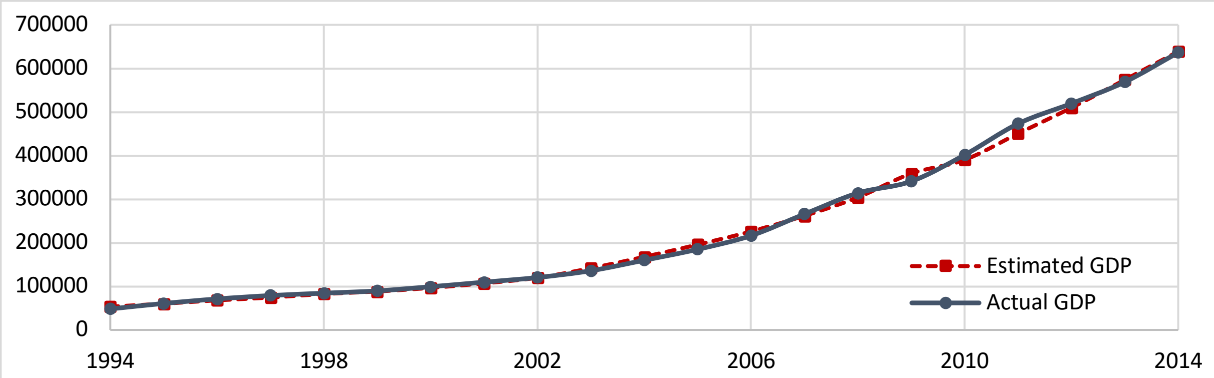

Figure 2. Actual GDP and Estimated GDP according to Ridge Regression.

Table 5.
Output elasticities for each input

Year	f_K	f_L	f_E
1994	0.51713	1.89821	0.08939
1995	0.52015	1.90034	0.09074
1996	0.52277	1.90213	0.09187
1997	0.52414	1.90317	0.09250
1998	0.52593	1.90469	0.09339
1999	0.52690	1.90534	0.09379
2000	0.52875	1.90662	0.09460
2001	0.53096	1.90820	0.09559
2002	0.53382	1.91026	0.09688
2003	0.53864	1.91358	0.09901
2004	0.54342	1.91683	0.10111
2005	0.54770	1.91989	0.10304
2006	0.55167	1.92273	0.10484
2007	0.55562	1.92563	0.10666
2008	0.55981	1.92867	0.10858
2009	0.56418	1.93201	0.11064
2010	0.56637	1.93354	0.11162
2011	0.57013	1.93631	0.11336
2012	0.57325	1.93868	0.11482
2013	0.57622	1.94093	0.11622
2014	0.57927	1.94289	0.11751
Average	0.54556	1.91860	0.10220

The output elasticities of variables of capital, labor and energy in Table 5 are given by years. We see that the output elasticity change with the time period t. These values represent that the output variables value fluctuation according to the sensitivity of input variables. Then, elasticities of substitution for each input factor are calculated and shown in Table 6.

In accordance with Table 6, the elasticity of substitution between the pairs of capital, labor and energy is positive. It means that these variables can be substitute. Technological development increases economic growth. Technological progress is described by the output mines the contribution of capital, labor and energy. We can calculate the contribution rate of technological progress to output growth as,

$$\alpha_A(t) = 1 - \alpha_K(t) - \alpha_L(t) - \alpha_E(t)$$

where,

$$\alpha_K(t) = f_K(t)\frac{\dot{K}_t}{K_t}\Big/\frac{\dot{Q}_t}{Q_t}, \quad \alpha_L(t) = f_L(t)\frac{\dot{L}_t}{L_t}\Big/\frac{\dot{Q}_t}{Q_t}, \quad \alpha_E(t) = f_E(t)\frac{\dot{E}_t}{E_t}\Big/\frac{\dot{Q}_t}{Q_t}.$$

In the investigated time period, the contribution rate of technological progress to total GDP growth is on average 3.52%.

Table 6.
Substitution elasticity of input factors

Year	σ_{KL}	σ_{KE}	σ_{LE}
1994	0.99396	0.99677	1.01225
1995	0.99389	0.99665	1.16627
1996	0.99382	0.99656	1.09318
1997	0.99379	0.99651	1.00486
1998	0.99375	0.99643	0.99975
1999	0.99373	0.99640	0.99975
2000	0.99368	0.99634	0.99975
2001	0.99363	0.99627	0.99975
2002	0.99356	0.99618	0.99975
2003	0.99344	0.99603	0.99975
2004	0.99332	0.99590	0.99975
2005	0.99322	0.99578	0.99975
2006	0.99313	0.99568	0.99975
2007	0.99303	0.99558	0.99975
2008	0.99293	0.99548	0.99975
2009	0.99283	0.99537	0.99975
2010	0.99278	0.99532	0.99975
2011	0.99269	0.99524	0.99975
2012	0.99262	0.99518	0.99975
2013	0.99255	0.99511	0.99975
2014	0.99247	0.99507	0.99976
Average	0.99328	0.99590	1.01297

In addition to the parameter estimations for capital, labor and labor input factors, the next period data of the current dataset belong to these input factors were forecasted with the help of Holt-Winter's model. Then, estimated capital, labor and energy data were compared with the actual data. See Figure3, Figure 4 and Figure 5 for relevant comparisons.

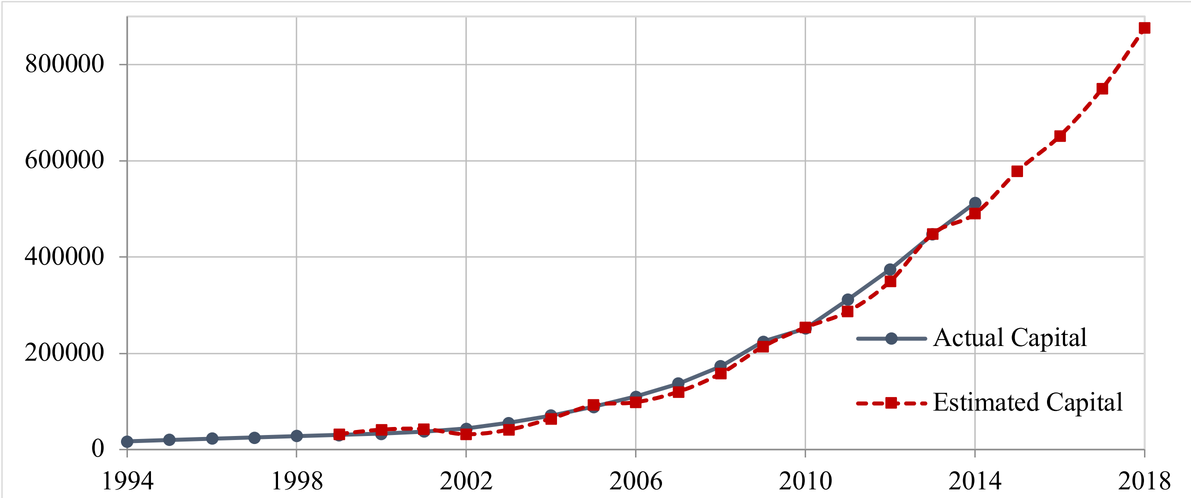

Figure 3. Estimated Capital according to Holt Winter's Method

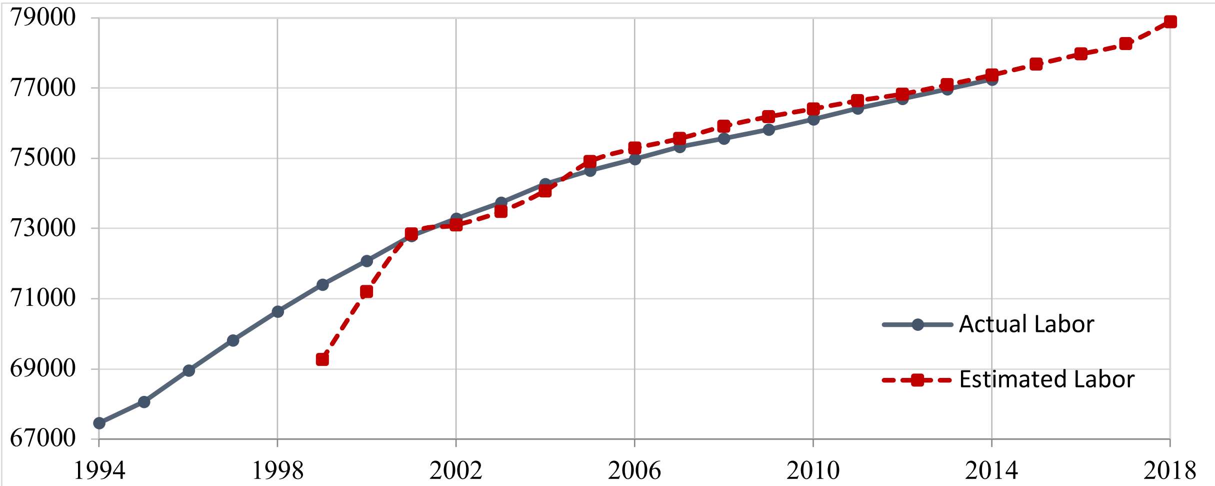

Figure 4. Estimated Labor according to Holt Winter's Method

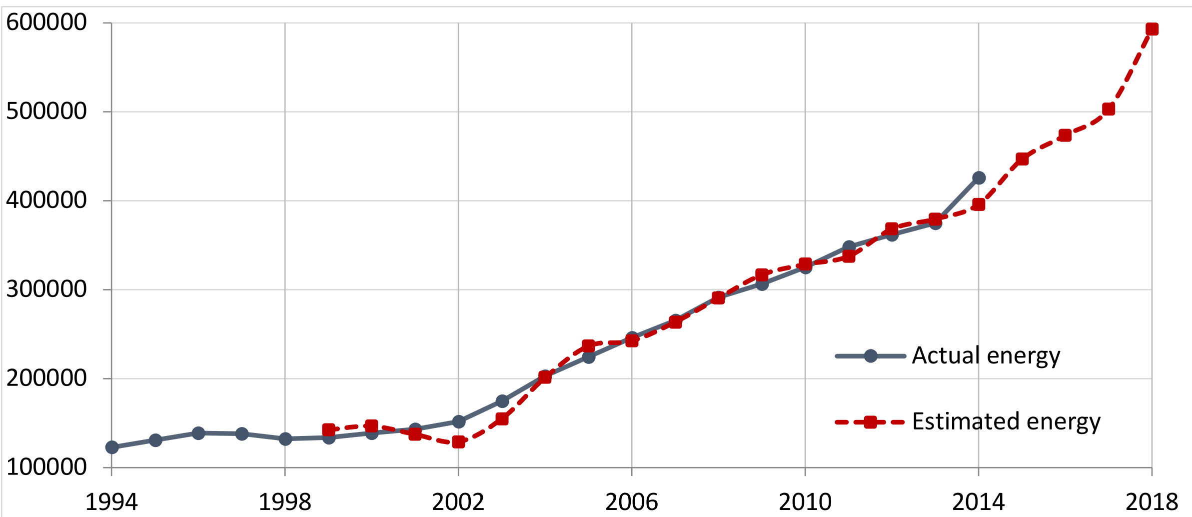

Figure 5. Estimated Energy according to Holt Winter's Method

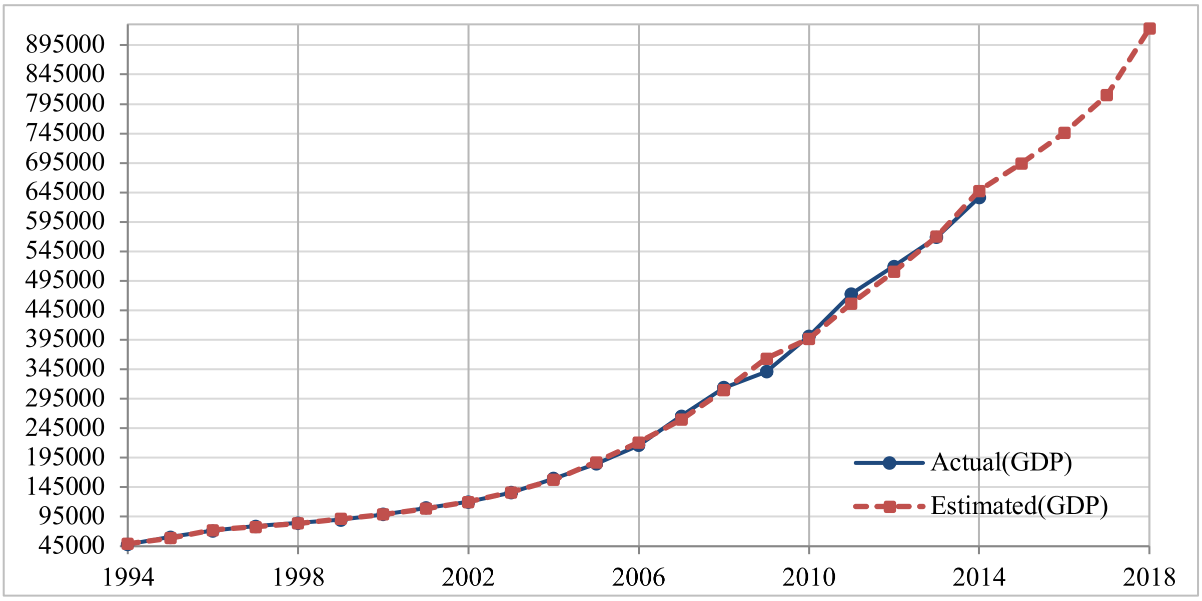

Figure 6. Estimated GDP according to Holt Winter's Method

The study showed that the triple exponential smoothing method can be an appropriate approach for level estimation of capital, labor and energy inputs. Here, as a control structure, we wanted to compare the GDP data we obtained with the results estimated by the IMF for the Translog production function. Table 7 was created for this purpose. When Table 7 is examined, it is seen that the estimation GDP results we obtained are quite close to the estimation obtained by the IMF.

Table 7.
Comparison with the estimated GDP by IMF and estimated GDP

Year	Estimated GDP (billion yuan)	Estimated GDP by IMF (billion yuan)	Actual GDP (billion yuan)
2012	510619.59	534123	519470.1
2013	570123.45	588018.8	568845
2014	570123.45	636139	636462.7
2015	694147.84	692391	
2016	745874.42	742082.3	
2017	810181.87	795409.2	
2018	922848.73	856923.6	

4.3.Error Analysis

Error Analysis is needed for examining the precision of forecasted results. The mean absolute percentage error (MAPE) is one of the most widely used methods which is evaluation of forecasting error, due to its advantages of scale-independency and interpretability (Kim & Kim, 2016). The MAPE can be defined as,

$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\frac{|Q_t - \hat{Q}_t|}{Q_t}.$$

Where Q_t is the actual value (Makridakis, Wheelwright, & Hyndman, 2008; Yıldırım & Başeğmez, 2016). $\hat{Q}_t$ is also forecasting value at time t. n is the number of periods forecasted. MAPE reference values for forecasting accuracy are shown in Table 8 (Lewis, 1982).

Accordingly, error analyses for both Translog and CES production functions were performed using MAPE and the relevant results are given in Table 9. According to Table 9, the MAPE value for the Translog production function is 3.39%, and the MAPE value for the CES production function is 3.31%. It can be said that the estimation process made in line with these values is highly accurate.

Table 8.
MAPE reference values for forecasting accuracy

MAPE	Forecasting Accuracy
Less than 10%	Highly accurate
11% to 20%	Good forecast
21% to 50%	Reasonable forecast
51% or more	Inaccurate forecast

Table 9.
MAPE values for Translog and CES production functions

	MAPE	Forecasting Accuracy
Translog production function	3.39%	Highly Accurate
CES production function	3.31%	Highly accurate

5. CONCLUSION

In this paper, CES and Translog production function model are implemented to estimate the future values of the GDP which is an indicator of China economic growth with capital, labor and energy input factors.

Firstly, we implemented the CES production model with capital and labor input factors to observed data. The findings of CES model reveal that the elasticity of substitution between capital - labor bundle is $\sigma = 1.085$ and MAPE value is 0.03423. The CES model with capital and labor inputs gives a reliable forecast of GDP, the nested CES production model can give more reliable results for large data set. Because we have a small number of data, we did not use the nested CES model in the real application.

Secondly, we implemented the quadratic Translog production function with capital, labor and energy inputs to the dataset. In the empirical analysis, it is found that multicollinearity between variables. To eliminate the multicollinearity, it is used to the Ridge regression method.

The Translog function is more convenient than CES function because it has a very flexible structure. The output elasticities of input factors which are changing by time are calculated by using the fitted model. Empirical findings show that Ridge regression parameters are consistent with the actual economic situation and Translog production function is a good explanatory model for China GDP values.

The output elasticity of each input factor and the elasticity of substitution between these input factors are analyzed. The output elasticities of capital, labor and energy are indicated that all of the input factors have a significant positive impact on China's economy in the studied period. The output elasticity of labor is slightly larger than the capital and energy elasticities. This also shows that labor input has a leading role in promoting economic growth. The fluctuation range of output elasticities of capital and labor is stable by years, however, the output elasticity of energy is gradually increasing by years. The contribution rate of technological progress to total GDP growth is found on average 3.52% in the investigated time period.

Finally, in general, the total production of the China economy highly depends on labor and capital inputs, besides the effect of energy input is relatively small.

There are also some limited cases of the study. The small number of data is the most important limitation. For this reason, the sensitivity of the estimation results obtained is debatable. However, it is difficult to reach data on especially capital and labor force in the long run. In future studies, more sensitive analyzes can be made by using machine learning techniques with more data.

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Competing interests
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